Background

The Ideal Gas Law. The Ideal Gas Law describes the relationship between the Pressure, Volume, Temperature, and the number of molecules in a gaseous sample. The statement of the law is as follows:

$$PV = nRT$$

where P is the pressure, V is volume, T is temperature in Kelvin, and n is the number of molecules in the sample. Here, R is the *ideal gas constant*. The value of R depends on the units of the other quantities. We will consider P in kPa, V in ml (equivalent to cm^3), T in K, and n in mmol. This gives R the following value and units:

$$R = 8.314\ \frac{kPa \cdot cm^3}{mmol \cdot K}$$

Internal Combustion Engines. A four stroke internal combustion engine consists of a cylindrical piston that fits snugly inside of a cylindrical chamber. An intake valve allows a gaseous fuel mixture to enter the chamber, and an exhaust valve allows post-combustion exhaust to exit the chamber. As the piston moves up and down, the volume of the chamber changes, and the pressure of gas inside changes according to the Ideal Gas Law. At the top of the chamber is a spark plug, which ignites the fuel mixture, causing an explosion that forces the piston downward, thus producing power. The four stages of the cycle are as follows:

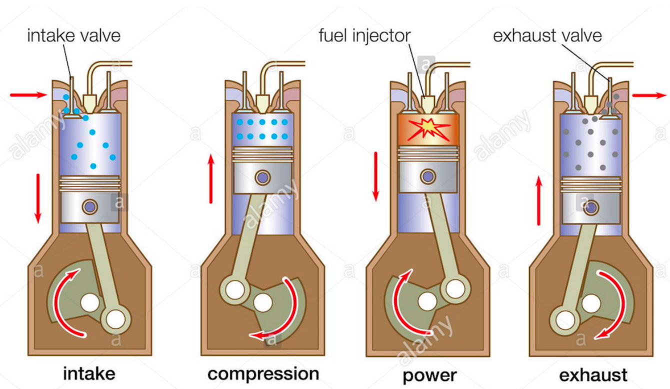

Figure 1: Diagram detailing the 4 stages of a 4-stroke internal combustion engine.

(1) Intake: The descending piston causes the effective volume of the chamber to increase. The intake valve is open, allowing fuel mixture to be sucked into the chamber.
(2) Compression: The intake valve closes. The piston ascends, decreasing the effective volume of the chamber, increasing the pressure of the fuel mixture.
(3) Ignition: A spark plug ignites the compressed fuel mixture at the top of the stroke. The combustion reaction causes both the temperature of the gas and the number of molecules to increase rapidly. This explosion, according to the Ideal Gas Law, causes the pressure and volume of the gas to increase, thus forcing the piston downward.
(4) Exhaust: As the piston ascends, the exhaust valve opens, allowing the products of the combustion reaction (mostly CO_2 and H_2O) to exit the chamber.

Problem Statement

Our cylindrical fuel chamber and piston have a radius of 2 cm. At the bottom of the stroke, when the volume is largest, the height of the cylindrical chamber is 10 cm. Recall the formula for the volume of a cylinder is $V = \pi r^2 h$. Assume that at the bottom of the stroke, when the cylinder is the largest, there is 5.61 mmol of fuel mixture in the chamber, with a temperature of 332 K and pressure of 123 kPa.

(a). Suppose that during the entire compression stroke, the piston is ascending at a rate of 5 $\frac{cm}{s}$. At what rate is the volume of the chamber decreasing during the compression stroke?

(b). Suppose that during compression the temperature of the gas remains constant and that the valves do not allow any gas to escape. Assuming the fuel mixture obeys the Ideal Gas Law, at what rate is the pressure of the gas increasing when the piston is halfway (5 cm) up the chamber?

(c). We now turn out attention to the moment the spark plug ignites the fuel. At this moment, the fuel will begin to burn, increasing the temperature of the gas *and* increasing the number of molecules in the gas, as the fuel is combusted into CO_2 and H_2O. Use logarithmic differentiation to differentiate both sides of the Ideal Gas Law. (This will give us the most "workable" form of the law when everything is changing).

(d). At the top of the stroke the piston is 1 cm from the top of the chamber. Suppose immediately after ignition, the temperature is increasing at 3320 $K \cdot s^{-1}$, the number of molecules is increasing at a rate of 16.83 mmol s^{-1} and the pressure is measured to increase at a rate of 1500 kPa s^{-1}. How fast is the volume of the chamber increasing?

(e). Given the rate at which the volume increases that you calculated in the previous part, how fast is the piston descending through the chamber due to the explosion?