

You may not use a graphing calculator or any communication device during the test. You may use a non-CAS, non-programmable calculator. Show all work in your blue book. Start each problem on a new page. Turn in this sheet inside your blue book; write your name, your row, room section, and test form on the front cover of the bluebook.

1. (28 pts) Determine whether the following series converge or diverge. Clearly state the tests you use and demonstrate how the series satisfies or fails the test.

$$(a) \sum_{n=3}^{\infty} \frac{1}{n^n} \quad (b) \sum_{n=1}^{\infty} \frac{e^n}{n^2} \quad (c) \sum_{n=2}^{\infty} \frac{n^3}{(n-1)^2(n^2+7)} \quad (d) \sum_{n=0}^{\infty} \frac{10^n}{n!}$$

2. (12 pts) Write the following series in sigma notation. Determine if the series converges and, if it does, find the sum of the series.

$$5 - 3 + \frac{9}{5} - \frac{27}{25} + \frac{81}{125} - \dots$$

3. (18 pts) Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n} \quad (b) \sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$$

4. (18 pts) Determine the interval and radius of convergence for the following power series.

$$(a) \sum_{n=1}^{\infty} \frac{n^2 x^n}{n!} \quad (b) \sum_{n=1}^{\infty} \frac{(x-2)^n}{2^n n^2}$$

5. (8 pts) Find a power series expansion for  $\frac{x^2}{1+x^3}$  and state its interval of convergence.

6. (16 pts) The power series expansion for  $\frac{1}{1+x^2}$  is given below:

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

Use this to find a power series expansion for  $\arctan(x)$ . Use the ratio test to determine the interval of convergence of the power series you find.

- Bonus. (5 pts) Use the power series you found for  $\arctan(x)$  to find a convergent series whose sum is  $\pi$ .